THERMAL CONDUCTIVITY OF PERMAFROST SOILS IN RELATION TO NATURAL MOISTURE CONTENT

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An experimental study is made of the thermal conductivity of permafrost soils within a broad range of natural moisture contents. Conductivity is calculated analytically in relation to the amounts of ice and unfrozen water and the minerological composition of the particles.

Calculation of the stability of structures in permafrost regions requires knowledge of the thermophysical properties of the frozen soil. In engineering practice, the study of these properties for a specific type of soil is usually performed as a function of the moisture content and the bulk weight of the skeleton. There is an unambiguous relationship between these parameters in the case of permafrost soils, since the latter are almost completely saturated with water in their natural state. Their air porosity is only 2-3% of their total volume [1]. Thus, when analyzing the thermophysical properties of permafrost soils, it is sufficient to consider just one of these parameters. The natural moisture content of the soil can be chosen as the governing parameter, since it is easily measured even under expeditionary conditions.

The total moisture content of frozen soils (particularly clayey soils) varies widely as a result of the migration of moisture during freezing–thawing and variation in sedimentation conditions. In particular, in the frozen alluvial deposits of Yakutia, the most likely values are 7-30% for sandy soils and 20-60% for sandy-loamy soils [1]. Here, the thermophysical properties of the soils may undergo substantial variations.

Figure 1 shows results of experimental studies of the thermal conductivity λ_0 of frozen alluvial soils in Yakutia within a broad range of saturation moisture contents w_0. In the

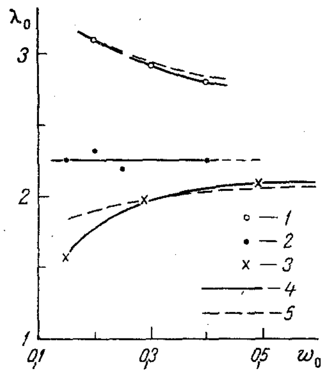

Fig. 1

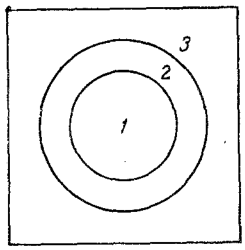

Fig. 2

Fig. 1. Dependence of the thermal conductivity λ_0, W/(m·K), of alluvial soils on the saturation moisture content in the frozen state: 1) sand; 2) sandy-loamy soil; 3) loam; 4) experimental curves; 5) theoretical curves.

Fig. 2. Three-component shell-like medium: 1) mineral skeleton of the soil; 2) unfrozen water; 3) ice.

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TABLE 1. Quantitative Content of Fractions in the Soils

Type of soil	Particle fractions		
	clayey	powdery	sandy
Sand	0,02	0,10	0,88
Sandy loam	0,06	0,30	0,64
Loam	0,20	0,37	0,43

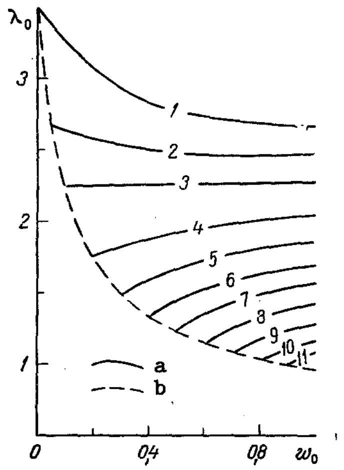

Fig. 3. Theoretical dependence of the thermal conductivity of soils λ_0 on saturation moisture content w_0 with λ_{sk} = 3.5 W/(m·K) and different contents of unfrozen water w_{uw}. Soils: a) frozen; b) thawed; 1) w_{bf} = 0; 2) 0.05; 3) 0.1; 4) 0.2; 5) 0.3; 6) 0.4; 7) 0.5; 8) 0.6; 9) 0.7; 10) 0.8; 11) 0.9.

TABLE 2. Parameters of Eq. (12) for Different Types of Soils

Type of soil	T_{bf}, K	ω^0_{vw}	A	a	b
Sand	272,8	0,080	0,076	4,375	—0,602
Sandy loam	272,8	0,136	0,062	1,190	—0,056
Loam	272,8	0,180	0,091	0,514	—0,013

experiments, the state of complete saturation of the soils with water was taken as the model of the natural state of permafrost soils. It is evident that the character of the dependence of λ_0 on w_0 is different for coarsely and finely dispersed soils. An increase in w_0 is accompanied by a decrease in thermal conductivity at the saturation limit for frozen sand, while the latter increases for loam and approaches the conductivity of ice. Sandy loams occupy an intermediate position.

The above features of the character of the relation $\lambda_0(w_0)$ can be attributed to the different contents of unfrozen water in the soils and differences in their mineralogical composition. In fine soils at the measurement temperature (about 263 K), the quantity of unfrozen water present is fairly substantial — on the order of 0.1. Thus, at low moisture contents, loam can be regarded as thawed ground. An increase in moisture content is accompanied by an increase in the role of ice inclusions in overall heat conduction by the soil, which leads to an increase in its thermal conductivity. In the case of a high moisture content, the thermal conductivity of loam approaches that of ice. The content of unfrozen water is negligible in sand, and the mineral particles are in direct contact with the ice. Since the mineral particles have agreater thermal conductivity than ice, then an increase in the content of moisture (ice) is accompanied by a reduction in the thermal conductivity of sand. The same mechanism is active in thawed soils. The phenomenon of the isolation of mineral particles by a film of unfrozen water is less in evidence in sandy soils than in loam. For the system mineral particle + unfrozen water, thermal conductivity evidently also takes the same values as for ice. Thus, frozen saturated sandy soil is characterized by nearly a constnt value of thermal conductivity throughout the range of variation of saturation moisture. The mineralogical composition of soils also plays an important role, so that the thermal conductivity of the mineral-bearing part of soil should increase in the transition from fine to coarse soils.

The above features of the thermal conductivity of permafrost soils can be studied quantitatively from the vantage point of the analytical theory of heat conduction of composite materials. Possible structural models of soils follows from the mechanism by which water is bound by mineral particles. Soil particles have excess surface energy, the latter depending on their fineness and mineralogical composition. When soils are moistened, molecular forces associated with the mineral particles cause the water to interact with the particles and surround the particles in concentric layers until extinction of the particles' excess surface energy. Particles of moistened soil, interacting with one another through a layer of bound water with a force not offset by the water, form a stable system of isolated particles. The rest of the pores are filled with free water. With a further decrease in temperature, new portions of the bound moisture begin to freeze. At low temperatures on the order of 253 K, almost no strongly bound water is frozen. In the frozen state, the system with isolated particles is strengthened by the ice that has already formed and becomes even more stable. Thus, to calculate the thermal conductivity of fine permafrost soils at different negative temperatures, it is possible to consider a three-component shell-like medium (mineral skeleton + unfrozen water + ice) in accordance with the scheme depicted in Fig. 2. The form of a sphere was taken for the mineral particles in this scheme.

The effective thermal conductivity of such a shell system λ_0 can be calculated by Maxwell's method on the basis of solution of the Laplace equation for a medium with a constant temperature gradient away from a spherical particle with a shell. Here, the theoretical formula has the form [2]

$$\frac{\lambda_0-\lambda_c}{\lambda_0+2\lambda_c}=\frac{v\left[\lambda_{uw}-\lambda_c-\dfrac{\varepsilon(\lambda_{uw}-\lambda_{sk})(2\lambda_{uw}+\lambda_s)}{2\lambda_{uw}+\lambda_c}\right]}{2\lambda_c+\lambda_{uw}+\dfrac{2\varepsilon(\lambda_{uw}-\lambda_{sk})(\lambda_c-\lambda_{uw})}{2\lambda_{uw}+\lambda_{sk}}}, \qquad (1)$$

where $\varepsilon=\frac{4\pi R_{sk}^3}{3}\Big/\frac{4\pi R_{uw+sk}^3}{3}$ is the volume fraction of the core of the particle; $v = 4n\pi R_{uw+sk}^3/3$ is the volume fraction of particles; n is the density of particles per unit volume of the mixture.

In Eq. (1), the parameters ε and v can be expressed through the relative content of mineral particles m_{sk} and unfrozen water m_{uw} over the entire volume of the system

$$\varepsilon=\frac{m_{sk}}{m_{sk}+m_{uw}} \text{ and } v=m_{sk}+m_{uw}.$$

Taking into account the relationship between m_{sk} and m_{uw} on the one hand and the saturation moisture content w_0 and quantity of unfrozen water m_{uw} on the other hand

$$m_{sk}=\frac{\rho_{sk}}{1+w_0\rho_{sk}} \text{ and } m_{uw}=\frac{\rho_{sk}w_{uw}}{1+w_0\rho_{sk}},$$

we obtain the following expression to calculate the thermal conductivity of moisture-saturated soil λ_0:

$$\lambda_0 = \lambda_c \frac{N+2M}{N-M}, \quad (2)$$

where

$$N = (1+w_0\rho_{sk})\left[2\lambda_s + \lambda_{uw} + \frac{2}{1+w_{uw}\rho_{sk}} \frac{(\lambda_{uw}-\lambda_{sk})(\lambda_c-\lambda_{uw})}{2\lambda_{uw}+\lambda_{sk}}\right], \quad (3)$$

$$M = (1+w_{uw}\rho_{sk})(\lambda_{uw}-\lambda_c) - \frac{(\lambda_{uw}-\lambda_{sk})(2\lambda_{uw}+\lambda_c)}{2\lambda_{uw}+\lambda_{sk}}. \quad (4)$$

All of the asymptotes are satisfied in Eqs. (2-4). At $w_0 \to \infty$, $\lambda_0 = \lambda_c$. If $w_0 = 0$ and $w_{uw} = 0$, then $\lambda_0 = \lambda_{sk}$. When $w_{uw} = 0$, we obtain the familiar Maxwell–Odelevskii formula for a two-component medium. In the expression for moisture content, this formula can be expressed in the form

$$\lambda_0 = \lambda_c \frac{(1+w_0\rho_{sk})(2\lambda_c+\lambda_{sk}) + 2(\lambda_{sk}-\lambda_c)}{(1+w_0\rho_{sk})(2\lambda_c+\lambda_{sk}) - (\lambda_{sk}-\lambda_c)}. \quad (5)$$

Equation (5) is applicable to thawed soils if we replace λ_c by the thermal conductivity of water λ_w.

The presence of captured air in frozen soils lowers their thermal conductivity. This can be expressed as follows:

$$\lambda_w = \frac{2\lambda_0(1+w\rho_{sk})}{2+\rho_{sk}(3w_0-w)}, \quad (6)$$

where w is the actual moisture content of the soil, which should vary within the range

$$w \geqslant \left(1 - \frac{0{,}4}{k}\right) w_0 - \frac{0{,}4}{\rho_{sk}}, \quad (7)$$

where k is a parameter dependent on the state of the soil and equal to unity for positive temperatures and 0.92 for negative temperatures.

Equation (6) can be useful in describing the thermal conductivity of thawed soil which has lost its excess water (in the absence of a water-confining stratum) and retains that amount of water determined by its water-confining capability.

To perform specific calculations, it is necessary to assign the thermal conductivities of the components of the soil in Eqs. (2)-(4). For pure ice at T = 273 K, λ = 2.25 W/(m·K) [3]. The thermal conductivity of unfrozen water can be taken approximately equal to that of free water, i.e, λ = 0.58 W/(m·K), since all of the anomalies in the properties of bound water pertain to its strongly-bound part — which is present in negligible quantities. The thermal conductivity of the mineral skeleton of the soil depends on the minerological composition of the particles and can be approximately evaluated by the formula:

$$\lambda_{sk} = \frac{1}{2}\left(\sum_{i=1}^{n} \lambda_i m_i + \frac{1}{\sum_{i=1}^{n} \frac{m_i}{\lambda_i}}\right). \quad (8)$$

Values of λ_i can be found in [4, 5].

The distribution of minerals in soils is determined by the conditions of sediment formation, which are extremely varied in nature. The amount of minerals in soils can be approximately calculated on the basis of the relation between the mineralogical and granulometric compositions of the particles. As is known, three fractions are distinguished in the granulometric classification of soil particles: clayey (<0.002 mm); powdery (0.002-0.05 mm), and sandy (0.05-2.0 mm). For practical purposes, it is assumed that the content of clay minerals —

particularly kaolinites — is equal to the content of clayey particles and 50% of the powdery particles [6]. The remainder of the soil is represented as consisting mainly of quartz and feldspars with an approximate quantitative ratio of 0.6:0.4.

The following values can be taken for λ_i, W/(m·K), in the calculations 6 for quartz; 2.5 for feldspars; 1.2 for kaolinite. The quantitative content of the fractions in the soils in accordance with Table 1.

Calculations by Eq. (8) with the above initial data result in the following approximate values of λ_{sk}, W/(m·K): 4.8 for sand; 3.3 for sandy loam; 2.7 for loam.

As an example, Fig. 3 shows the results of calculations of the thermal conductivity of permafrost soils by Eq. (2) with λ_{sk} = 3.5 W/(m·K). It is evident that this formula for the most part accurately reflects the character of the dependence of the thermal conductivity of permafrost soils on the saturation moisture content and the quantity of unfrozen water. As might be expected, the content of unfrozen water in the soils plays the deciding role in the character of the dependence of λ_0 on w_0.

Depending on the quantity of unfrozen water, this dependence becomes decreasing (for sands with w_{uw} = 0) or increasing (for fine soils). The value of λ_0 nearly becomes independent of w_0 at a certain value of w_{uw}, which is typical of sandy loams. The lower boundary of the dependences of λ_0 on w_0 shown in Fig. 3 is the thawed state of the soils when they are saturated.

If the initial data used in the proposed method of calculation is the known thermal conductivity of a moist soil for some saturation moisture content — which is desirable for soils in their densest states — then it is not necessary to evaluate the thermal conductivity of the mineral skeleton of the soil.

Let us examine this question in greater detail. We will use the three-component shell-like model of the medium shown in Fig. 2. In contrast to the previous case, here the central component 1 is moist soil whose thermal conductivity has been previously measured (λ_0^{ini}) at the saturation moisture content w_0^{ini}. In frozen soils, this moisture is present as a film of unfrozen water around spherical particles. The comopnent of the shell 2 is that part of the unfrozen water that is not part of w_0^{ini}. On the whole, component 3 consists of ice. Given this formulation of the problem, the parameters of Eq. (2) can be written as follows:

$$N = 2\lambda_c + 2\lambda_w + \frac{2(\lambda_w - \lambda_0^{ini})(\lambda_c - \lambda_w)}{2\lambda_w + \lambda_0^{ini}} \frac{\rho_w(k + \rho_{sk} w_0^{ini})}{\rho_w(k + \rho_{sk} w_0^{ini}) + k\rho_{sk} w'_{uw}}, \tag{9}$$

$$M = \frac{1}{k + \rho_{sk} w_0} \Big\{ (\lambda_w - \lambda_c)[\rho_w(k + \rho_{sk} w_0^{ini}) + k\rho_{sk} w'_{uw}] - \frac{(k + \rho_{sk} w_0^{ini})(\lambda_w - \lambda_0^{ini})(2\lambda_w + \lambda_c)}{2\lambda_w + \lambda_0^{ini}} \Big\}, \tag{10}$$

where w'_{bf} is the excess unfrozen water.

If the total content of unfrozen water (w_{uw}) is less than the initial saturation moisture content w_0^{ini} of the soil or is equal to zero, then there is no excess unfrozen water (w'_{uw} = 0). If the content of unfrozen water is greater than the initial saturation moisture content of the soil, then $w'_{uw} = w_{uw} - w_0^{ini}$.

Let us examine the limiting cases.

1. Let there be no excess unfrozen water in the soil (w'_{uw} = 0). The unfrozen water in the soil, along with the ice, enter into the initial saturation moisture content w_0^{ini}. We then obtain

$$\lambda_0 = \lambda_c \frac{2\lambda_c(w_0 - w_0^{ini}) + \lambda_0^{ini}(w_0 + 2w_0^{ini} + g)}{\lambda_c(2w_0 + w_0^{ini} + g) + \lambda_0^{ini}(w_0 - w_0^{ini})}, \tag{11}$$

where $g = 3k/\rho_{sk}$.

This formula can also be applied to thawed soils if we replace λ_i by the thermal conductivity of water λ_w.

2. At $w_0 = w_0^{ini}$, $\lambda_0 = \lambda_0^{ini}$.

3. Let $w_0 \to \infty$. Then $\lambda_0 = \lambda_c$ or $\lambda_0 = \lambda_w$, i.e., the thermal conductivity of the soil approaches the thermal conductivity of ice or water at large values of saturation moisture content.

Thus, in Eqs. (9) and (10), all of the limiting cases are satisfied. In these formulas, the initial thermal conductivity of the soil λ_0^{ini} should correspond to the saturated state of the soil. If the degree of moistening is insufficient, then a correction can be made to the initial parameter in accordance with Eq. (6).

Comparison of the calculated data with the experimental data (see Fig. 1) shows that Eq. (11) provides satisfactory results, especially for sand and sandy loam. The data diverges for loam at low moisture contents. It may be that the condition of complete saturation of the specimens was not satisfied in the experiments. Some of themicropores are evidently not filled at low moisture contents ($\approx$15%), and captured air causes the measured thermal conductivity of the loam to be somewhat understated.

In conclusion, we note that the amount of water in soils depends on the temperature T and is described by the formula of N. S. Ivanov [7]:

$$w_{uw} = w_{uw}^0 + A\left[\frac{1}{1 + a(T - T_{bf}) + b(T - T_{bf})^2} - 1\right], \quad (12)$$

where T_{bf} is the temperature corresponding to the beginning of freezing of the soil moisture; w_{uw}^0 is the quantity of unfrozen water at T_{bf}; A, a, and b are parameters dependent on the type of soil (Table 2).

NOTATION

λ, thermal conductivity; m, volumetric content of soil components; w, weight moisture content; ρ, specific weight; T, temperature. Indices: 0, state of maximum water saturation; w, moist state; sk, skeleton of soil; uw, unfrozen water; c, ice; w, water; i, i-th species of rock and mineral in the mineral particles of the soil; ini, initial state; bf, beginning of freezing of pore moisture.

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